AIEEE 2012

Part -B Mathematics

31. **Statement 1:** The sum of the series $1+(1+2+4)+(4+6+9)+(9+12+16)+....+(361+380+400)$ is 8000

Statement 2: $\sum_{k=1}^{n}\left(k^3-(k-1)^3\right)=n^3$, for any natural number n.

(1) Statement-1 is True, Statement-2 is False.

(2) Statement-1 is False, Statement-2 is True.

(3) Statement-1 is True, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(4) Statement-1 is True, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

Ans: [3]

32. An ellipse is drawn by taking a diameter of the circle $(x-1)^2+y^2=1$ is its semiminor axis and a diameter of the circle $x^2+(y-2)^2=4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is:

(1) $x^2+4y^2=16$ **(2)** $4x^2+y^2=4$

(3) $x^2+4y^2=8$ **(4)** $4x^2+y^2=8$

Ans: [1]

33. The length of the diameter of the circle which touches the x-axis at the point $(1,0)$ and passes through the point $(2,3)$ is:

(1) $5/3$ **(2)** $10/3$

(3) $3/5$ **(4)** $6/5$

Ans: [2]

34. Let P and Q be 3×3 matrices with $P\neq Q$. If $P^3=Q^3$ and $P^2Q=Q^2P$, then determinant of (P^2+Q^2) is equal to:

(1) -1 **(2)** -2

(3) 1 **(4)** 0

Ans: [4]

35. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is

(1) a rational number other than positive integers

(2) an irrational number

(3) an odd positive integer

(4) an even positive integer

Ans: [2]

36. **Statement 1:** An equation of a common tangent to the parabola $y^2=16\sqrt{3}x$ and the ellipse $2x^2+y^2=4$ is $y=2x+2\sqrt{3}$

Statement 2: If the line $y=mx+\frac{4\sqrt{3}}{m}$, $(m\neq0)$ is a common tangent to the parabola $y^2=16\sqrt{3}x$ and the ellipse $2x^2+y^2=4$, then m satisfies $m^4+2m^2=24$

(1) Statement-1 is True, Statement-2 is False.

(2) Statement-1 is False, Statement-2 is True.

(3) Statement-1 is True, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(4) Statement-1 is True, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

Ans: [3]

37. Three numbers are chosen at random without replacement form $\{1,2,3,...,8\}$. The probability that their minimum is 3, given that their maximum is 6 is:

(1) $\frac{2}{5}$ **(2)** $\frac{3}{8}$

(3) $\frac{1}{5}$ **(4)** $\frac{1}{4}$

Ans: [3]

38. If $g(x)=\int_0^x \cos 4t\,dt$ then $g(x+\pi)$ equals:

(1) $g(x).g(\pi)$ **(2)** $\frac{g(x)}{g(\pi)}$

(3) $g(x)+g(\pi)$ **(4)** $g(x)-g(\pi)$

Ans: [3]

39. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is

(1) 879 **(2)** 880

(3) 629 **(4)** 630

Ans: [1]

40. If 100 times the 100th term of an AP with non zero common difference equals the 50 times its 50th term, then the 150th term of this AP is

(1) zero **(2)** – 150

(3) 150 times its 50th term **(4)** 150

Ans: [1]

41. The area bounded between the parabolas $x^2=\frac{y}{4}$ and $x^2=9y$, and the straight line $y=2$ is

(1) $10\sqrt{2}$ **(2)** $20\sqrt{2}$

(3) $\frac{10\sqrt{2}}{3}$ **(4)** $\frac{20\sqrt{2}}{3}$

Ans: [4]

42. An equation of the plane parallel to the plane $x-2y+2z-5=0$ and at a unit distance from the origin is

(1) $x-2y+2z+5=0$ **(2)** $x-2y+2z-3=0$

(3) $x-2y+2z+1=0$ **(4)** $x-2y+2z-1=0$

Ans: [2]

43. The equation $e^{\sin x}-e^{-\sin x}-4=0$ has

(1) exactly four real roots.

(2) infinite number of real roots.

(3) no real roots

(4) exactly one real root.

Ans: [3]

44. The negation of the statement

"If I become a teacher, then I will open a school", is :

(1) I will not become a teacher or I will open a school

(2) I will become a teacher and I will not open a school

(3) Either I will not become a teacher or I will not open a school.

(4) Neither I will become a teacher nor I will open a school.

Ans: [2]

45. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt}=0.5\,p(t)-450$. If $p(0)=850$, then the time at which the population becomes zero is:

(1) ln 18 **(2)** 2 ln 18

(3) ln 9 **(4)** 1/2 ln 18

Ans: [2]

46. If the integral

$$\int \frac{5\tan x}{\tan x-2}dx = x + a\ln|\sin x-2\cos x|+k$$

then a is equal to:

(1) 2 **(2)** –1

(3) –2 **(4)** 1

Ans: [1]

47. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} + 4\hat{b}$ are perpendicular to each other, then the angule between $\hat{a}$ and $\hat{b}$ is:

(1) $\frac{\pi}{4}$ **(2)** $\frac{\pi}{6}$

(3) $\frac{\pi}{2}$ **(4)** $\frac{\pi}{3}$

Ans: [4]

48. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the roigin. If the area of the triangle OPQ is least then the slope of the line PQ is:

(1) $-\frac{1}{2}$ **(2)** $-\frac{1}{4}$

(3) -4 **(4)** -2

Ans: [4]

49. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is

(1) 5^3 **(2)** 5^2

(3) 3^5 **(4)** 2^5

Ans: [3]

50. Let ABCD be a parallelogram such tht $\vec{AB} = \vec{q}$, $\vec{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed form the vertexd B to the side AD, then $\vec{r}$ is given by:

(1) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{q})}\vec{p}$ **(2)** $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{q})}\vec{p}$

(3) $\vec{r} = -\vec{q} + \frac{(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{q})}\vec{p}$ **(4)** $\vec{r} = \vec{q} - \frac{(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{q})}\vec{p}$

Ans: [3]

51. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals:

(1) $11/5$ **(2)** $29/5$

(3) 5 **(4)** 6

Ans: [4]

52. In a ΔPQR, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to:

(1) $\frac{3\pi}{4}$ **(2)** $\frac{5\pi}{6}$

(3) $\frac{\pi}{6}$ **(4)** $\frac{\pi}{4}$

Ans: [3]

53. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column materices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to

(1) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ **(2)** $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(3) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ **(4)** $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

Ans: [1]

54. If $f:R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is

(1) continuous only at $x = 0$

(2) continuous for every real x

(3) discontinuous only at $x = 0$

(4) discontinuous only at non-zero integral values of x

Ans: [2]

55. A spherical balloon is filled with 4500 π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72 π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is

(1) $9/2$ **(2)** $9/7$

(3) $7/9$ **(4)** $2/9$

Ans: [4]

56. Let $a, b \in R$ be such that the function f given by $f(x) = \ln|x| + bx^2 + ax$, $x \neq 0$ has extreme values at $x = -1$ and $x = 2$.

Statement 1: f has local maximum at $x = -1$ and at $x = 2$.

Statement 2: $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.

(1) Statement-1 is True, Statement-2 is False.

(2) Statement-1 is False, Statement-2 is True.

(3) Statement-1 is True, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(4) Statement-1 is True, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

Ans: [3]

57. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies:

(1) on the imaginary axis

(2) either on the real axis or on a circle passing through the origin.

(3) on a circle with centre at the origin.

(4) either on the real axis or on a circle not passing through the origin.

Ans: [2]

58. Consider the function, $f(x) = |x-2| + |x-5|, x \in R$

Statement 1: $f'(4) = 0$

Statement 2: f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$

(1) Statement-1 is True, Statement-2 is False.

(2) Statement-1 is False, Statement-2 is True.

(3) Statement-1 is True, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(4) Statement-1 is True, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

Ans: [4]

59. Let $x_1, x_2, ...x_n$ be n observation, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.

Statement 1: Variance of $2x_1, 2x_2, ...2x_n$ is $4\sigma^2$

Statement 2: Arithmetic mean of $2x_1, 2x_2, ...2x_n$ is $4\bar{x}$

(1) Statement-1 is True, Statement-2 is False.

(2) Statement-1 is False, Statement-2 is True.

(3) Statement-1 is True, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(4) Statement-1 is True, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

Ans: [1]

60. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to :

(1) 0 **(2)** -1

(3) $\frac{2}{9}$ **(4)** $\frac{9}{2}$

Ans: [4]